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## Maximizing Consensus in Portfolio Selection in Multicriteria Group Decision Making

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### Abstract

This paper deals with a scenario of decision making where a moderator selects a (sub)set (aka portfolio) of decision alternatives from a larger set. The larger the number of decision makers who agree on a solution in the portfolio the more successful the moderator is. We assume that decision makers decide independently from each other but indicate their preferences with respect to different objectives in terms of desirability functions, which can be interpreted as cumulative (probability) density functions. A procedure to select a solution with maximal expected number of decision makers that accept it is provided. Moreover, this is generalized to sets of solutions. An algorithm for computing and maximizing the expected number of decision makers that can agree on at least one solution in a subset of decision alternatives is developed. Computational aspects, as well as practical examples for using this for item selection from a database will be discussed.

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**Keywords:** Multicriteria Decision Analysis, Multiobjective Optimization, Desirability Functions, Poisson Binomial Distribution, Consensus, Group Decision Making.

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## 1. Introduction

The problem of selecting interesting subsets of solutions from a larger set of solutions is a problem that is recently often discussed in multi-criteria decision making [4,21,22,23,24]. In previous work it was suggested to model the preference of the decision maker by means of desirability functions [5,6] and include these as utility functions in heuristic search for optimal subsets [1]. The idea is to map objective function values to a value between zero and one, where zero means that this objective function value is not acceptable, and one means that the objective function value has reached a fully acceptable level. In [23] it is suggested to interpret the desirability functions as cumulative density functions (CDFs). In the case of independent objective functions and an independent consideration of these by the DMs, the product of desirability functions leads to the probability that a solution is accepted. Moreover for a set of decision alternatives they computed the probability that at least one solution in the set (portfolio) is accepted. The approach was further detailed and generalized to group decision making in [24].

In this work we deal with a similar approach. Again we consider a posteriori multi-criteria decision making. But this time the focus is on the *expected number of decision makers who accept a solution*. Again, the approach requires the decision makers to specify acceptance thresholds or desirability functions for different objective functions and they can then be used to select a small subset of  $k$  solutions from a larger set of  $n$  solutions, e.g., from a database, and present this set to  $q$  decision makers. The consensus of a solution is then the number of decision makers, which accept that solution. Given that the acceptance decision is modeled stochastically, also the consensus of a solution is a stochastic variable.

The paper will start with the introduction of the probabilistic model (Section 2). Then, in Section 3, the new concepts of consensus and expected consensus are introduced. These are contrasted to the previously proposed acceptance probability [24]. Thereafter, in Section 4, the definition and the computation of the consensus and expected consensus for a set of solutions is discussed.

## 2. Preliminaries and Description of Data

We consider the following scenario:

- A group of  $q$  decision makers, denoted by  $DM_j, j = 1, \dots, q$ .
- A large set of  $n$  decision alternatives (e.g. products, projects, etc.), from which a moderator wants to preselect a small subset of  $k$  solutions to present to the decision makers who then have to agree on at least one solution from the subset.
- For each solution the  $m$  objective function values  $f_i(x_l), i \in \{1, \dots, m\}, l \in \{1, \dots, n\}$  are known. They are either to be minimized or to be maximized.
- For each decision maker and objective a desirability function  $D_i^j: \mathbb{R} \rightarrow [0,1], i \in \{1, \dots, m\}, j \in \{1, \dots, q\}$  is specified. A value of zero refers to rejection, a value of 1 to acceptance, and values between 0 and 1 define the probability of acceptance. The function is monotonically increasing (decreasing) in case of maximization (minimization).

These data are used to assess the performance of solutions (subsets of solutions) with respect to the probability of acceptance of the solution (at least one solution). In addition we look at the performance of subsets of solutions with respect to the probability that there is a solution in the subset on which all decision makers agree, or with respect to the expected number of agreements that can be achieved for some solution in the subset (expected consensus).

Desirability functions of the Derringer Suich type [5] will be used here. They are intuitive to understand and require only three parameters, a minimum threshold  $DMIN$ , a maximum threshold  $DMAX$ , and a curvature  $r$ . In case of maximization  $DMIN$  is the level of the objective function value  $f$  below which a solution will be rejected,  $DMAX$  is the level above which the objective function value will be definitely accepted. Between these levels the function increases monotonically and its curvature is governed by  $r$ . The desirability function is defined piecewise:

$$D(f) = \begin{cases} 0 & \text{if } f \leq DMIN \\ \left( \frac{f - DMIN}{DMAX - DMIN} \right)^r & \text{if } DMIN < f \leq DMAX \\ 1 & \text{otherwise} \end{cases}$$

For minimization we can also define a desirability function. Values above  $DMAX$  will be rejected and values below  $DMIN$  will be definitely accepted. From  $DMIN$  to  $DMAX$  the function decreases monotonically.

$$D(f) = \begin{cases} 1 & \text{if } f \leq DMIN \\ \left( \frac{DMAX - f}{DMAX - DMIN} \right)^r & \text{if } DMIN < f \leq DMAX \\ 0 & \text{otherwise} \end{cases}$$

In Figure 1 we provide an example of two desirability functions. On the left hand side a desirability function for maximization is plotted.  $DMIN$  is 1 and  $DMAX$  is 3. On the right hand side a desirability function for minimization is plotted,  $DMIN$  is 60 and  $DMAX$  is 75. Clearly the left hand side shows the graph of a non-negative, non-decreasing function and  $\lim_{x \rightarrow \infty} f(x) = 1$ . In other words it has all the defining properties of a CDF (see [23, 24] for more details on the relationship between desirability functions and CDFs).

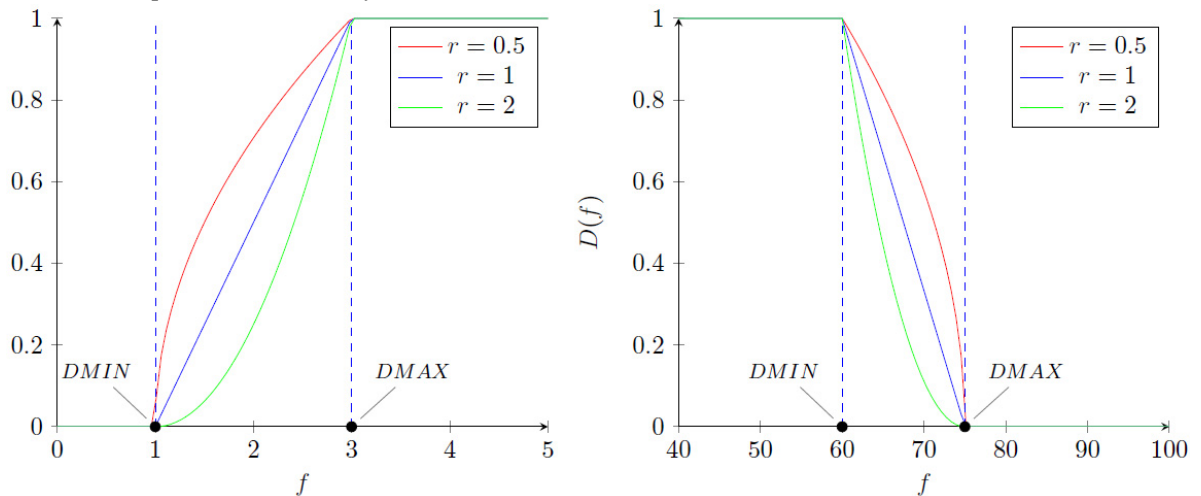


Figure 1. Derringer Suich desirability functions with different curvatures  $r$  for maximization (left) and minimization (right).

### 3. Computation of Consensus and Acceptance Probability

The goal is to provide a subset (aka *portfolio*) of bounded size that maximizes the expected maximum of the consensus achieved among its solutions. It is assumed that the choice of the decision makers is guided by  $m$  criteria (aka *objective functions*). For each objective function  $f_i$ ,  $i \in \{1, \dots, m\}$  and each decision maker  $DM_j$ ,  $j \in \{1, \dots, q\}$  it is known or estimated *a priori* what the acceptance probability for that criterion in isolation is. This probability will be denoted with  $P_i^j(x)$ , where  $x$  is a point in the database. In case of minimization this probability is given by the probability that a function value at or below a certain threshold, say  $\tau$ , will be accepted. This can be modeled by a Cumulative Distribution Function (CDF)  $D_i^j: \mathbb{R} \rightarrow [0, 1]$  with  $P_i^j(x) = D_i^j(f_i(x))$ .

The CDFs are elicited *a priori* by questioning the DMs. We use Derringer Suich Desirability Functions. If we assume that the objective functions are independent and more importantly in the decision process they are treated

independently by the DMs, then we can compute the probability that a decision maker  $DM_j$  accepts a solution  $x$  by the following product:

$$\Pr\left(\bigwedge_{i=1}^m (DM_j \text{ accepts the } i\text{-th objective function value of } x)\right) = \prod_{i=1}^m D_i^j(f_i(x))$$

In previous work [24] the probability that *all* decision makers accept the solution  $x$  was computed for independent decision makers as

$$\Pr\left(\bigwedge_{j=1}^q \bigwedge_{i=1}^m (DM_j \text{ accepts the } i\text{-th objective function value for } x)\right) = \prod_{j=1}^q \prod_{i=1}^m D_i^j(f_i(x))$$

Now, given a set  $X$  of  $k$  solutions  $X = \{x_1, \dots, x_k\}$ , the probability that *all* decision makers agree on at least one solution is computed in [24] by

$$\Pr\left(\bigvee_{l=1}^k \bigwedge_{j=1}^q \bigwedge_{i=1}^m (DM_j \text{ accepts the } i\text{-th objective function value for } x_l)\right) = HI(D(X))$$

Here  $D(X) = \{d\{x_1\}, \dots, d\{x_k\}\}$  is a set of  $m$  dimensional vectors. These are given by

$$d_i(x_l) = \prod_{j=1}^q D_i^j(f_i(x_l)), i \in \{1, \dots, m\}, l \in \{1, \dots, k\}$$

and  $HI(Y)$  denotes the hypervolume indicator [20] of a set  $Y \subset \mathbb{R}^m$  for a reference point of  $(1, \dots, 1)^T \in \mathbb{R}^m$ :

$$HI(Y) = \text{Vol}_m\left(\bigcup_{l=1}^k [y_l, 1]\right)$$

Here  $\text{Vol}_m$  is the Lebesgue measure on  $\mathbb{R}^m$ . In 2-D it is the area, in 3-D the volume, and in  $m$ -D the hypervolume of the measured set.

A problem with this scenario is that when one decision maker has a zero probability of acceptance on only one of the objective functions of a solution, this solution is already assigned a zero probability of acceptance by all decision makers. This makes it very likely that a single, demanding, decision maker dominates the choice process by being too demanding on a single criterion. Solutions that have a zero probability will appear indifferent to other solutions with a zero probability and it is not seen whether one has some advantages w.r.t. other solutions. Given these preliminaries we can be more detailed about the remaining discussion: First we will study how to compute the expected number of decision makers that can agree on a single solution -- or on at least one solution in a given portfolio of size  $k$ . The discussion will include computational aspects. Then we will ask the question of how to find the set of solutions of size  $k$  from a larger set of  $n$  solutions that maximizes this expectation (Section 4). This problem can only be solved heuristically and we will discuss a genetic algorithm that searches for this set. Finally we will discuss potential implementations and applications, both in multi-objective optimization and in database selection (Section 5 and 6).

#### 4. Computation of Expected Number of Decision Makers to Accept a Solution or a Portfolio

The expected number of decision makers that accept a single solution  $x$  can be computed as follows: Assume that each decision maker can either vote 'pro' or 'against' the solution. The probability that  $DM^j$  selects 'pro' is  $P^j = \prod_i P_i^j(x) = \prod_i D_i^j(f_i(x))$ . We can model the random vector of acceptances by the decision makers as  $c = (c_1, \dots, c_q) \in \{0, 1\}^q$  and compute the expected value of decision makers to vote 'pro' as the sum of the probability of events times the sum of 'pros' in an event over all events. Here an event is a single voting pattern  $c$ :

$$E(x) = \sum_{c \in \{0, 1\}^q} \left( \prod_{j=1}^q (c_j P^j(x) + (1 - c_j)(1 - P^j(x))) \right) \cdot \sum_{j=1}^q c_j$$

Here  $\prod_{j=1}^q (c_j P^j(x) + (1 - c_j)(1 - P^j(x)))$  is the probability for the event  $\mathbf{c}$  to occur, and  $\sum_{j=1}^q c_j$  is the value of the event. The equation above is equal to a sum of Bernoulli independently distributed random variables, also known in the literature as a *Poisson Binominal Distribution*. It is known that the expected value of such a distribution can be computed as the sum of probabilities, i.e.:  $E(x) = \sum_{j=1}^q P^j(x)$ . This is the expected number of decision makers who will agree on  $x$ . It is easy to find the solution, which maximizes this score. From the theory of the Poisson Binominal Distribution also the variance is known:  $V(x) = \sum_{j=1}^q (1 - P^j(x))P^j(x)$ .

Another situation emerges if we can compute (and select) a subset of solutions and present it to the DMs. In this case we need to compute the expected value of the number of DMs who accept the solution with the maximal number of received acceptances. For a given set of  $k$  solutions we will call this number the expected consensus ( $EK(X)$ ). More precisely, let us denote the number of decision makers accepting a solution  $x$  with  $A(x)$ . Moreover let us define the consensus of a set  $X = \{x_1, \dots, x_k\}$  as  $K(X) = \max\{A(x_1), \dots, A(x_k)\}$ . Then, given the acceptance of a solution is a probabilistic event governed by the distributions  $D_i^j$ , the expected consensus for a set of solutions  $X$  is defined as  $\sum_{a=0}^q a \Pr(K(X) = a)$ , and denoted by  $EK(X)$ . Note that  $A(\cdot)$  and  $K(\cdot)$  are now random variables.

To the best of our knowledge, to compute the expected value precisely is very difficult.

A Monte Carlo simulation of  $K$  is however straightforward, and can be accomplished by the following algorithm:

**Algorithm 1** Monte Carlo Computation of  $EK(X)$

```

    Input: Acceptance Probability Distributions  $D_i^j: \mathbb{R} \rightarrow [0,1], i = 1, \dots, m, j = 1, \dots, q;$ 
           Set of solutions  $X = \{x_1, \dots, x_k\}$ 
    Output:  $EK(\{x_1, \dots, x_k\})$ 
1    $s \leftarrow 0$ 
2   REPEAT  $MAX\_IT$  Times
3        $A[1] \leftarrow 0, \dots, A[k] \leftarrow 0$  // Counter for number of acceptance
4       FOREACH  $i \in \{1, \dots, m\};$ 
5           FOREACH  $j \in \{1, \dots, q\}$ 
6                $\tau_i^j \sim D_i^j$  // Choose  $\tau_i^j$  according to distribution  $D_i^j$ 
7           FOREACH  $l \in \{1, \dots, k\}$ 
8               FOREACH  $j \in \{1, \dots, q\}$ 
9                   IF  $\bigwedge_{i=1}^m (f_i(x_l) \geq \tau_i^j)$  THEN  $A[l] \leftarrow A[l] + 1$  //in minimization:  $f_i(x_l) \leq \tau_i^j$ 
10           $K \leftarrow \max_{l \in \{1, \dots, k\}} A[l]$ 
11           $s \leftarrow s + K$ 
12  Return  $s/MAX\_IT$ 

```

This simulation assumes that decisions are independent. Moreover, if one solution is accepted also another solution is accepted if it Pareto dominates the first one. This is why the decision thresholds  $\tau_i^j$  stay fixed in the loop from line 7 to line 9. Also, it is important here to assume that there exist true decision thresholds, but they are modeled probabilistically because the true thresholds are not known. In this sense the probabilities should be understood as Bayesian belief probabilities. The number of iterations should be such that the estimation value stabilizes.

## 5. Computational Example

Consider the following problem. A group wants to book hotel rooms in the same hotel. They have to decide on a choice of a hotel. Star rating and price per room are the two objectives to be considered, the former is to be maximized and the latter minimized. The star rating and the price per room are provided in a database (partially shown in Table 2). For a given subset, of say 3 solutions, the expected consensus has to be computed. Consider the following example data (Table 1 and 2):

Table 1: Decision makers' preferences table

Decision maker	<i>DMIN</i> Stars	<i>DMAX</i> Stars	<i>DMIN</i> Price	<i>DMAX</i> Price
DM1	1	3	60	75
DM2	2	4	60	80
DM3	4	5	70	100
DM4	2	3	50	90
DM5	4	5	50	75

Table 2: Decision alternatives and their objective function values

Solution/Portfolio of 3	Stars	Price
Hotel1	4	60
Hotel2	3	50
Hotel3	5	90

In this example  $q = 5$ ,  $n = 3$ ,  $m = 2$ . The desirability functions are chosen as piecewise linear functions. For an example, see Figure 1( $r=1$ ). The expected consensus is computed by means of simulation using Algorithm 1 for 100 iterations ( $MAX_{IT}$ ). The expected consensus is  $EK = 2.85$  with standard deviation 0.35887 (see Table 3). This means that we can expect 2 – 3 decision makers to agree on at least one hotel in the set. Note that there is one decision maker in the set, DM5, who is extremely demanding, to the extent that s/he will reject all solutions with probability 1. If we would, like in [24] assess the performance of the portfolio by means of the probability that one solution is accepted by *all* decision makers, we would get a probability of 0.

If we want to suggest only a subset of two hotels, we can compute the following table:

Table 3: Subsets of decision alternatives of size 2 and their expected consensus and the standard deviation of it

Subset	$EK$	$\sigma(K)$
Hotel1, Hotel2	2.85	0.35887
Hotel1, Hotel3	2.77	0.422953
Hotel2, Hotel3	2.45	0.5

From the result, it is clear that, in case that we would like to present only two hotels to the group of decision makers, it would be best to choose Hotel 1 and Hotel 2 (see Table 3), because it has the highest expected consensus and smallest standard deviation. Finally, we may ask the question what would be the solution (subset of size 1) to choose when only one solution should be suggested to the decision makers. Here we can use the precise formula (Table 4):

Table 4: Single decision alternatives and their expected consensus (with exact value in the parenthesis) and the standard deviation of it.

Solution	$EK$	$\sigma(K)$
Hotel1	2.77 (exact 2.75)	0.422953
Hotel2	2.45 (exact 2.5)	0.5
Hotel3	0.32 (exact 0.33)	0.468826

Clearly, we will not present Hotel 3, because it is likely that no decision maker will agree on it. The most attractive choice is Hotel 1, which has the highest expected consensus and the smallest standard deviation.

As we chose a piecewise linear desirability function it is possible to compute the probability of acceptance of a single solution for a single decision maker by means of:

$$P_l^j = \prod_{i=1}^m \min(f_i(x_l), DMAX_i^j) - \min(f_i(x_l), DMIN_i^j) / \prod_{i=1}^m DMAX_i^j - DMIN_i^j$$

The same can be done for minimization (cost), by replacing the *min* in the above formula by *max*. Here the values of  $P_l^j$ , i.e. the probability that decision maker  $j$  accepts solution  $l$  are computed in Table 5.

Table 5: Data used for the computation of the expected consensus of single solutions

	$l$	$DMINstar$	$DMAXstar$	$DMINprice$	$DMAXPrice$	Stars	Price	$P_l^j$
$DM^1$	1	1	3	60	75	4	60	1
	2	1	3	60	75	3	50	1
	3	1	3	60	75	5	90	0
$DM^2$	1	2	4	60	80	4	60	1
	2	2	4	60	80	3	50	0.5
	3	2	4	60	80	5	90	0
$DM^3$	1	4	5	70	100	4	60	0
	2	4	5	70	100	3	50	0
	3	4	5	70	100	5	90	0.333333
$DM^4$	1	2	3	50	90	4	60	0.75
	2	2	3	50	90	3	50	1
	3	2	3	50	90	5	90	0
$DM^5$	1	4	5	50	75	4	60	0
	2	4	5	50	75	3	50	0
	3	4	5	50	75	5	90	0

## 6. Practical implementation in database systems

The new way to assess performance can now readily be integrated in database queries or in decision support tools. For the first we imagine a modification of the skyline query, which could look as follows:

```
SELECT R.ID, R.A1, ..., R.Am FROM MyTable R WHERE
    <GOAL1l>(R.A1,l1,u1,1), ..., <GOALml>(R.Am,lm,um,1),
    <GOAL1q>(R.A1,l1,u1,q), ..., <GOALmq>(R.Am,lm,um,q) BOUND k,
```

where  $GOALmq$  denotes the parameters of the desirability function of the  $m$ -th goal of the  $q$ -th DM. The query is stated by the moderator after collecting information from the DMs. In case GOAL is replaced by the keyword MIN, minimization is used, in case GOAL is replaced by MAX maximization is used, and in case GOAL is replaced by TARGET and desirability function for minimizing distance to target is considered and  $lm$  is interpreted as the target value. R.ID is the ID of the solution (which could be accompanied by any other attribute of the solution that we might find interesting).

An example would be to select from a database of hotels for some holiday destination, say 'Faro', the one that most likely satisfies the two DMs, of which one likes to be close to the city and another one wants to be in a more quiet remote location, away from the city center:

```
SELECT H.id H.name H.price, H.distance_to_city, H.rating FROM Hotels H WHERE
    H.city='Faro' AND
    MIN(H.price,100,40,1), MAX(H.rating,2,1,3),
    MIN(H.price,75,30,2), MAX(H.rating,3,5,2) BOUND 7.
```

This query selects a portfolio of maximally 7 hotels (id, name, price, distance\_to\_city, rating) aiming to present at least one solution that satisfies the two DMs in terms of price (first DM: 40 – 100 Euro, second DM: 30-75 Euro), and rating (first DM: 1-3stars, second DM 3-5 stars).

In future we plan to implement queries like this and collect user feedback for different application domains, such as drug discovery, engineering, cyber security, or online catalogues. Also, we want to answer if more parameters should be used for modelling the acceptance probabilities, for instance for the curvature or correlations between objectives and decision makers' choices. Also, it is possible to use the query in a more automated procedure, for instance in an application that asks the DMs for their preferences and then outputs the information on acceptance probabilities in a visual way. The solutions in the portfolio with the highest individual acceptance probabilities can be displayed first.



It would also be advantageous to have the possibility in SQL like systems to order the above query output by considering also the standard deviation as a second criterion, or another risk criterion.

In addition it will be interesting to compare to alternative probabilistic models for decision making, such as models from financial portfolio theory that recently found their application in set-oriented search [25], and to compare different modelling approaches, such as alternative desirability functions by Harrington [6] which are strictly positive.

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